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Extrapolation of optimal lifted system ILC solution, with application to a waferstage.

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Abstract

The optimal solution from the lifted system design methods presented by Tousain [1] has many advantages over classic *Iterative Learning Control* (ILC) design methods, with one drawback: the ILC solution has to be recalculated for every trajectory of a different length. This paper shows that it is very well possible to avoid this by expressing the solution for the lifted system ILC as a finite time function of the plant parameters. This solution will be referred to as an extrapolated solution since the solution can be used for a trajectory of any length without recalculating the solution. This low order extrapolated solution has been compared to a high order lifted system optimal control (Q-)ILC solution in the application to an industrial grade wafer stage, showing the value of this extrapolated solution.

1 Introduction

In many servo control application the same task is repeatedly performed in the same way. Iterative learning control has been shown to be a very effective way to obtain control signals to greatly reduce the errors during these tasks by using the knowledge of similarity between the tasks ([2],[3],[4],[5],[6],[7]). In 1988 Phan and Longman [8] presented a setting describing the design of an ILC as a finite time problem where the input and output vectors of a plant are considered as discrete finite vectors. This setting is known as a lifted system description. Some research has been done towards reducing the calculation complexity of the lifted system ILC design problem through the use of basis functions [9] but it has only recently been shown that classical feedback design methods can quite easily be applied to the lifted system description [1]. In ILC design methods that are based on infinite time considerations (like transfer functions [2]) the nonzero value of the error to be used by the ILC at the start and end of the trajectory (which can be caused by system noise), causes problems that need to be handled separately often resulting in rather heuristic approaches. Design in the finite time lifted system setting has as a main advantage that the solution explicitly takes into account states of the plant at the beginning and end of the trajectory, resulting in a time varying operation that can be applied without any extra effort.

One drawback however is that these design methods are based on a fixed size of the lifted system, which essentially means that the ILC will be calculated for a fixed length of the trajectory under consideration and the application to a trajectory of different length will change the design and require a recalculation of the ILC. This paper shows that it is possible to design an optimal control ILC independently of the length of the trajectory, purely based on the low order plant parameters, resulting in a low order flexible solution with the advantages of an optimal control design.

[1]This research project is being sponsored by Philips CFT

2 ILC Design setting

In this paper the lifted system representation as presented in e.g. [8] and [1] will be used. In this representation the system dynamics are considered as a static map, which describes the systems behaviour along a finite time interval. This is illustrated in figure 1 where H is a matrix describing the finite time input-output map of the plant under consideration, in this case a wafer stage (P) stabilized by a PID controller[2] (C). The input and output to this system are vectors with length N, corresponding to the number of samples in the time interval under consideration. S denotes the sensitivity of the plant and H is the process sensitivity:

$$H = \frac{P}{1+PC} \quad S = \frac{1}{1+PC} \tag{1}$$

The trajectory to be followed $\hat{y}_{ref}$ is a vector that is constant from trial to trial so it can be considered as a constant disturbance that needs to be suppressed. The internal model prin-

[2]for an LTI system H will be a lower triangular Toeplitz Matrix containing the impulse response coefficients of the LTI system.

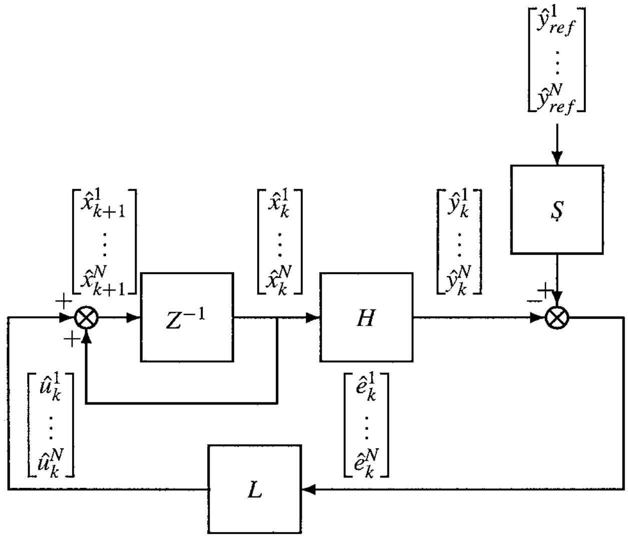

Figure 1: Stacked System ILC setting.

ciple [10] now states that a model of this disturbance should be present in our feedback controller to be able to suppress this disturbance, in this case an integrator. Note that both the constant disturbance and this integrator are considered within the *trial*-domain, i.e.: Z^{-1} denotes *1 trial* delay. The discrete time integrator in figure 1 is in fact a bank of N integrators.

The ILC controller is denoted with L in figure 1, which is the feedback interconnection between the error signal e_k and the input of the system x_k. The system we now consider is a multivariable system given by:

$$\begin{aligned} \hat{x}_{k+1} &= \hat{x}_k + \hat{u}_k \\ \hat{e}_k &= -H\hat{x}_k + \hat{y}_{ref} \\ \hat{u}_k &= L\hat{e}_k \\ \hat{u}_0 &= \hat{z}_0 \end{aligned} \tag{2}$$

this clearly shows that the states of the ILC system are x_k, and the objective of the ILC (L), will be to stabilize this system which is equivalent to convergence of the ILC in a classical sense. Any L that stabilizes the ILC system will eliminate all systematic errors if the number of trials go to infinity.

Any feedback design method can be used for the design of L, but in general a trade-off between the output performance and the ILC-system states (x_k) is preferable, yielding a trade-off between non systematic noise amplification and convergence (stability) of the ILC. A particularly charming method was presented in [1], where a constraint on the noise amplification and a quadratic objective are taken into consideration in a multi objective design. For clarity, in this paper only the results for an LQR design are discussed, with a linear quadratic objective (3) to balance the inputs and outputs of the system:

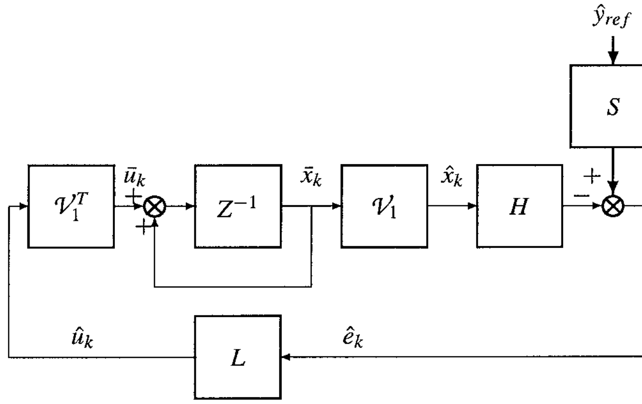

Figure 2: lifted system with extra $\mathcal{V}_1$.

$$\begin{aligned} J &= \sum_{k=1}^{\infty} \hat{y}_k^T Q' \hat{y}_k + \hat{u}_k^T R \hat{u}_k \\ &= \sum_{k=1}^{\infty} \hat{x}_k^T H^T Q H \hat{x}_k + \hat{u}_k^T R \hat{u}_k \end{aligned} \tag{3}$$

The solution to this problem can be found by finding the solution to the discrete time algebraic Riccati equation, but the system tends to get very large if long trajectories are considered so it may be a numerically intensive procedure.

One must note that the output matrix H may be a singular or nearly singular matrix, which will happen if the underlying plant contains delays or non-minimum phase zeros (see [1] for details). In this case the system will not be fully observable, which will be a problem for an optimal control solution. To solve this define the singular value decomposition of H, where $\mathcal{U}_1$ and $\mathcal{V}_1$ are unitary orthogonal matrices and Σ is a diagonal matrix with the singular values on the main diagonal ordered from large to small:

$$\begin{aligned} H = \mathcal{U}\Sigma\mathcal{V}^T \qquad & V^T V = U^T U = I \\ \Sigma = diag(\sigma_1..\sigma_N) \qquad & \sigma_1 > \sigma_2 ... > \sigma_N \\ H = \begin{pmatrix} \mathcal{U}_1 & \mathcal{U}_2 \end{pmatrix} & \begin{pmatrix} \Sigma_1 & 0 \\ 0 & \Sigma_2 \end{pmatrix} \begin{pmatrix} \mathcal{V}_1^T \\ \mathcal{V}_2^T \end{pmatrix} \end{aligned} \tag{4}$$

Σ_2 contains the d singular values from Σ which are (nearly) zero, so H can be written as:

$$H \approx \mathcal{U}_1 \Sigma_1 \mathcal{V}_1^T \tag{5}$$

$\mathcal{U}_1$ and $\mathcal{V}_1^T$ will in general not be square.

$\mathcal{V}_1^T$ and $\mathcal{V}_1$ can now be added to the system as shown in figure 2, resulting in an effective controller of lower order than the size of P, which only affects the observable part of the system. Addition of these 2 matrices also enables the solution of the LQR problem as a set of scalar equations as shown below.

3 ILC Design method

With the transformation:

$$\begin{aligned}\hat{u}_k &= \mathcal{V}_1 \bar{u}_k \\ \hat{x}_k &= \mathcal{V}_1 \bar{x}_k\end{aligned} \tag{6}$$

system (2) will become [3]:

$$\begin{aligned}\bar{x}_{k+1} &= \bar{x}_k + \bar{u}_k \\ \hat{e}_k &= -H\mathcal{V}_1\bar{x}_k + \hat{y}_{ref} \\ \bar{u}_0 &= \mathcal{V}_1 \hat{z}_0 \\ \bar{u}_k &= \bar{L}\hat{e}_k\end{aligned} \tag{7}$$

with

$$\bar{L} = \mathcal{V}_1^T L \tag{8}$$

The objective in equation 3 can be rewritten to:

$$J = \sum_{k=1}^{M} \hat{x}_k^T \mathcal{V}_1^T P^T Q P \mathcal{V}_1 \hat{x}_k + \hat{u}_k^T \mathcal{V}_1^T R \mathcal{V}_1 \hat{u}_k \tag{9}$$

Choosing $Q = I$ and $R = \beta I$ will enable the exactly tuning of the balance between the inputs and outputs with the parameter β. Substitution of equation 5 then yields:

$$J = \sum_{k=1}^{M} \bar{x}_k^T \Sigma_1^2 \bar{x}_k + \beta \bar{u}_k^T \bar{u}_k \tag{10}$$

with the solution given by:

$$Q - S\mathcal{V}_1^T\left(\beta I + \mathcal{V}_1 S \mathcal{V}_1^T\right)^{-1}\mathcal{V}_1 S = 0 \tag{11}$$

$$LP\mathcal{V}_1 = \left(\beta I + \mathcal{V}_1 S \mathcal{V}_1^T\right)^{-1}\mathcal{V}_1 S \tag{12}$$

which gives us the feedback interconnection matrix L:

$$\begin{aligned}L\mathcal{U}_1\Sigma_1 &= \mathcal{V}_1\left(\beta S^{-1} + I\right)^{-1} \\ L &= \mathcal{V}_1\left(\beta S^{-1} + I\right)^{-1}\Sigma_1^{-1}\mathcal{U}_1^T\end{aligned} \tag{13}$$

This shows that L will be an adjusted inverse of the plant P, and this adjustment is exactly caused by the weighting introduced with β. Above equations are fully diagonal, so the solution to the Riccati equation (11) will also be diagonal which can be solved elementwise. With σ_i and s_i, denoting the i-th elements of Σ_1 and S:

$$s_i = \frac{1}{2}\sigma_i^2\left(1 + \sqrt{1 + \frac{4\beta}{\sigma_i^2}}\right) \tag{14}$$

For illustration of the approximation discussed in the next section it is shown that this can be approximated by a Taylor expansion with:

$$s_i \approx \sigma_i^2 + \beta \tag{15}$$

To illustrate the structure of the optimal control ILC design,

[3]The vectors with an overbar are in general shorter than the regular vectors. See equation 5

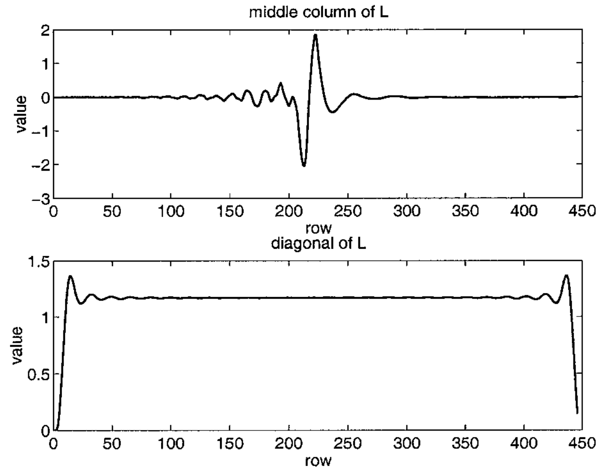

Figure 3: structure of L

the main diagonal and the middle column are shown in figure 3, of a small (450×450) designed L. This matrix can be seen as the impulse response matrix containing the Markov parameters of L. The plot of the middle column illustrates that the underlying system for L is not causal, since it contains non-zero Markov parameters above the diagonal. The diagonal shows that the Markov parameters vary in time, although this variation is mainly located at the start and end of the trajectory; the diagonal is constant in the middle of the trajectory. This time varying behavior at the start and the end of the trajectory set it apart from infinite time based design methods. In the next section a design is presented describing this non-causal time varying behavior of the optimal control ILC with the underlying low-order plant parameters.

4 Upscaled (low order) ILC design method

The solution from the previous section can in general be calculated for long trajectories too, but that will require a large singular value decomposition to obtain $\mathcal{V}_1$. For long trajectories, and hence large matrices, this is not practical. In this section a design is presented that describes the optimal ILC solution in the parameters of the underlying LTI system, making calculations (singular value decompositions) on the full impulse response matrices unnecessary.

For systems with delays or non-minimum phase zeros, H will be singular or nearly singular matrix as mentioned above. Rather than using the singular value decomposition of equation (5), it is possible to select the observable part of H using a non-square identity matrix, since the number of (nearly) zero valued singular values (denoted with d) is independent on the size of H:

$$\tilde{I}^T = \begin{bmatrix} I & 0 \end{bmatrix} \tag{16}$$

Where $\tilde{I}$ is a $N \times (N-d)$ matrix. Using these matrices as a replacement of $\mathcal{V}_1$ in figure 2 is effectively the same as the removal of the last columns of H as presented in [1]. In implementation this means that the measurement horizon is d samples longer than the control signal. New output and feedback matrices can now be defined:

$$\begin{aligned} \tilde{L} &= \tilde{I}^T L \\ \tilde{H} &= H\tilde{I} \end{aligned} \tag{17}$$

Now consider the solution for the optimal control problem stated in (3) with $Q = I$ and $R = \beta I$:

$$J = \sum_{k=1}^{M} x_k^T \tilde{H}^T \tilde{H} x_k + \beta u_k^T u_k \tag{18}$$

The corresponding Riccati equation is:

$$-S(\beta I + S)^{-1} S + \tilde{H}^T \tilde{H} = 0 \tag{19}$$

and the feedback interconnection:

$$\begin{aligned} \tilde{L}\tilde{H} &= S^{-1}\tilde{H}^T\tilde{H} \\ \tilde{L} &= S^{-1}\tilde{H}^T \end{aligned} \tag{20}$$

The solution to the Riccati equation (19) can be approximated in a very similar way to equation (15) yielding:

$$S = \tilde{H}^T \tilde{H} + \beta I \tag{21}$$

Substitution of this approximate solution in the Riccati equation shows that it is a solution to an optimal control problem with slightly different weighting $\tilde{Q}$. Since it still is the solution of an optimal control problem ($\tilde{Q} > 0$), it still yields a stable solution and hence a convergent ILC:

$$\tilde{Q} = \tilde{H}^T \tilde{H} + \beta^2 I \left(\tilde{H}^T \tilde{H} + 2\beta I\right)^{-1} \tag{22}$$

Note that the solution of the Riccati equation for $\tilde{Q}$ approaches the solution for Q for small values of β.

As seen in the previous section the matrices L and S can become extremely large when long trajectories are considered. However, for an LTI system, H is a finite time transfer function of the underlying LTI system, so its effect can be simulated with the system description:

$$\begin{aligned} x(t+1) &= Ax(t) + Bu(t) \\ y(t) &= Cx(t) \end{aligned} \tag{23}$$

Using this notion, the effect of equation (21) can be calculated over any time interval without actually having to calculate the full matrices corresponding to that time interval. The appropriate equations are two linked difference equations, one with a forward (causal) recursion, and one with a backward (anti causal) recursion. The LTI system equations describing equation (21) are:

$$\begin{aligned} x(l+1) &= Ax(l) + Bu(l) \\ \tilde{u}(l) &= Cx(l) \\ p(l) &= A^T p(l+1) + C^T \tilde{u}(l) \\ y(l) &= B^T p(l) + \beta u(l) \end{aligned} \tag{24}$$

In equation (20) the inverse of this relation is needed which can be found by expressing the input $u(t)$ as a function of the output $y(t)$:

$$\begin{aligned} x(l+1) &= Ax(l) + Bu(l) \\ p(l) &= A^T p(l+1) + C^T Cx(l) \\ u(l) &= \beta^{-1} y(l) - \beta^{-1} B^T p(l) \end{aligned} \tag{25}$$

This is a Hamiltonian equation, which can be decoupled into two difference equations with one equation containing the stable eigenvalues (Z_{11}), which can be simulated forwards in time, and one containing unstable eigenvalues (Z_{22}), which can be simulated backwards in time:

$$\begin{bmatrix} z(l) \\ \tilde{z}(l) \end{bmatrix} = T^{-1} \begin{bmatrix} x(l) \\ p(l) \end{bmatrix} \tag{26}$$

$$\begin{aligned} \begin{bmatrix} z(l+1) \\ \tilde{z}(l+1) \end{bmatrix} &= \begin{bmatrix} Z_{11} & 0 \\ 0 & Z_{22} \end{bmatrix} \begin{bmatrix} z(l) \\ \tilde{z}(l) \end{bmatrix} + T^{-1} \begin{bmatrix} \beta^{-1} B \\ 0 \end{bmatrix} y(l) \\ u(l) &= \beta^{-1} y(l) - \beta^{-1} B^T \left(T_{21} z(l) + T_{22} \tilde{z}(l)\right) \end{aligned} \tag{27}$$

Simulating these equations over the desired trajectory while taking into account the start and end conditions yields the exact response of matrix $\tilde{L}$ in equation (20). For more details details about this decoupling please contact the author.

The effect of $\tilde{H}^T$ in equation 20 can be included in the same way with the addition of another anti-causal difference equation. This solution again shows that the optimal ILC feedback interconnection is a mixed causal/anti-causal operation, which is typical for an ILC design. Since a measurement for the full trial is available for the ILC this is not a problem.

5 Experimental Setup

The method described above has been used to eliminate oscillations in one direction of a high precision wafer stage as shown in figure 4. This machine consists of a large granite base with a moving table on top of it (the chuck) that is actuated by linear electro-motors. The granite base is fitted with compliant dampers to the ground to isolate it from outside disturbances. The measurement system consists of laser-interferometers which measure the position of a reflective surface on the sides of the table. This experimental setup is very similar to the modern wafer stages in use in IC production, which shows the relevance for industrial application.

The system has force inputs and position outputs so the system consists of a double integrator, making it open-loop unstable. Therefore the system is stabilized with a PID controller, and this closed loop system is then considered as plant for the ILC. Figure 5 shows the process sensitivity H and the sensitivity S of the plant. The sample frequency of the system is 5000Hz.

During the lithographic process of IC manufacture the chuck containing a silicon disk is repeatedly moved from

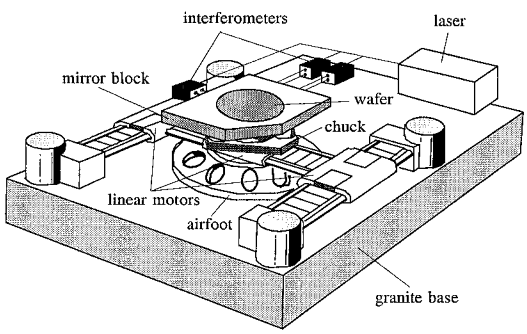

Figure 4: Wafer stage of a wafers stepper
picture is shown with courtesy of D.de Roover[2] and Philips CFT

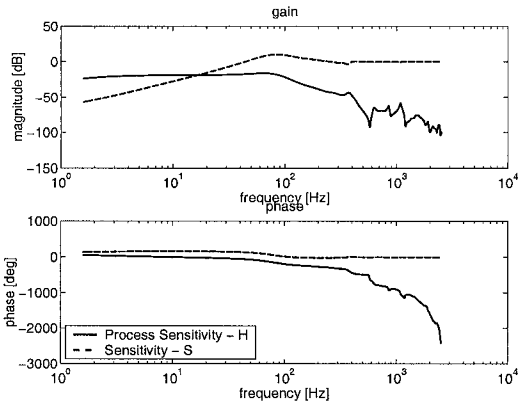

Figure 5: Bode plot of wafer stage setup.

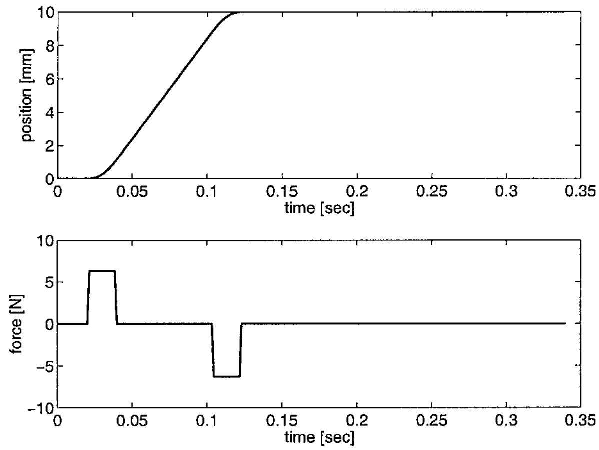

Figure 6: trajectory with force feedforward

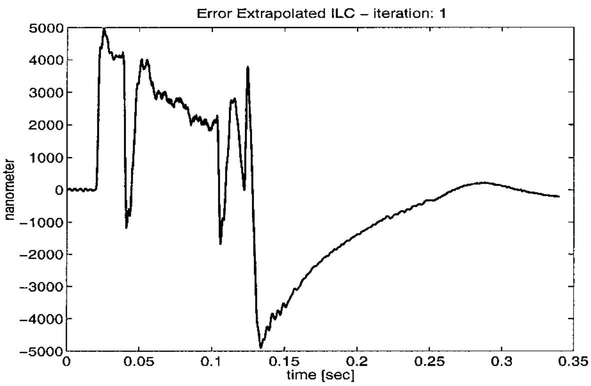

Figure 7: tracking error at trial 1

one position to the next position to expose each part of the silicon disk. The trajectory considered is a 3rd order profile describing a single one of such steps of the wafer stage. Figure 6 shows the reference trajectory and the force feedforward signal. The ILC will focus on adjusting the feedforward signal to obtain the smallest error possible with respect to the reference trajectory.

Figure 7 shows the tracking error for the first trial corresponding to the feedforward shown in figure 6. The feedforward resulting from the ILC procedure after 15 trials is shown in figure 8, clearly showing the anti-causal anticipation behaviour that is so typical for ILC. The actuation of the system starts before the actual trajectory starts, resulting in a great reduction of the tracking error. Both the large scale and the upscaled design methods resulted in nearly identical feedforward signals so only one is shown. In figure 9 the tracking error is shown at trial 15 for the full LQILC solution and figure 10 shows the same error for the extrapolated solution. We can see that both ILC designs result in a error reduction of a factor 30, leaving only non-systematic noise and high frequencies in the resulting error.

The trajectory used in this example was 1750 samples long, which would result in a feedback matrix L with 3.062.5000 elements each stored at double precision, resulting in 25Mb of memory usage for just the feedback matrix. This is a relatively short trajectory, but already the size of the needed matrices can be prohibiting in implementation. The system matrices A,B,C,D describe a 33th order model, requiring 1155 parameters to be stored, requiring 9240bytes of storage. For practical implementation this difference is very significant and especially for longer more realistic trajectories it is essential for implementation of these designs.

6 Conclusions

In this paper it has been reasoned that the optimal Iterative Learning Control design in the lifted system description poses a practical problem when using the design methods for large trajectories. Since the length of the trajectory does not change the system behaviour it has been shown that it is possible to calculate the Linear Quadratic Optimal ILC in a way that makes the solution independent of the length of the

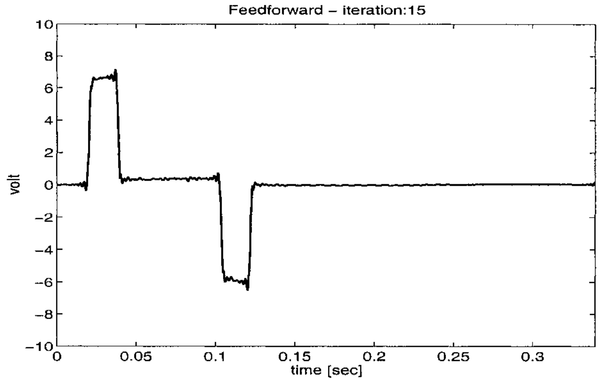

Figure 8: force feedforward at trial 15

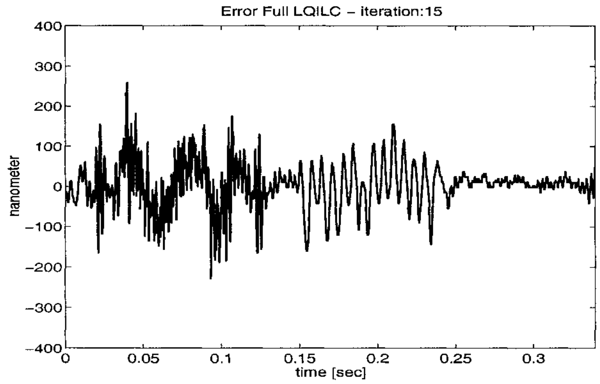

Figure 9: error after 15 trials with full LQILC

trajectory, as is the case in many classical ILC design methods ([2],[11],[3],[5]). This extrapolated solution has been compared with the full LQILC design in an application on a wafer stage, showing the same performance for both solutions. This extrapolation method shows that it is very well possible to make trajectory independent finite time ILC designs based on classical feedback design methods without having to resort to any heuristic tricks to make the designs work in a practical application.

It is recommended that the lifted system setting is used for any ILC design method, since it captures all the dynamics involved in the design of an ILC, and as shown has no real disadvantages compared to classical ILC design methods. It does however facilitate understanding and analysis of Iterative Learning Control.

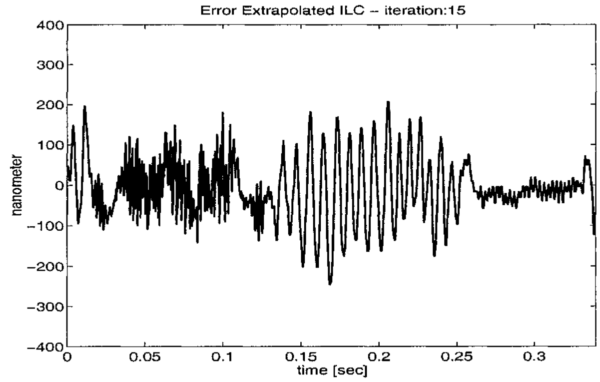

Figure 10: error after 15 trials with extrapolated LQILC

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