

# Noise suppression in buffer-state Iterative Learning Control, applied to a high precision wafer stage.

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## Abstract

Iterative Learning control has been proven to be very effective at suppressing repetitive errors in broad spectrum of applications. Ever increasing demands on performance in positioning systems have led to the need for more advanced control systems, like ILC, to achieve the desired performance. A major drawback of any ILC is that in general it will amplify any noise present in the measurement of the system which, as will be show in this paper, can lead to undesired *loss* of performance.

In this paper such a basic ILC [1] is compared to a buffer-state ILC [2] with respect to the noise amplification, with application to a wafer stage. The advantages of the buffer-state based ILC design, over basic ILC design in handling system noise are illustrated by measurements on a wafer stage, clearly showing the advantage of using full buffer-state feedback to optimise a combination of both the ILC convergence rate and the ILC noise amplification.

## 1 Introduction

In many Iterative Learning Control applications the goal is to achieve very high performance, even with limited system knowledge. ILC is very well suited for eliminating systematic tracking errors, it is however known to amplify system noise which can limit the obtained increase in performance. This paper illustrates that it is very well possible to design an ILC taking into account the full state of the memory loop, in such a way that the convergence speed is not compromised but noise amplification is kept to a minimum.

Any control system design is limited by the system model available to it. For ILC this is no different, the better the model, the better ILC performance will be (although for a near perfect model ILC may actually be

redundant). There are several 'model-free' ILC methods available but for a fair comparison both the basic ILC method and the new ILC design method discussed in this paper are based on the same model.

First the basic ILC design method will be briefly discussed, followed by a slightly more extensive discussion of the buffer-state optimal ILC design. After that several measurements will be discussed for different ILC design settings.

## 2 Experimental wafer-stage

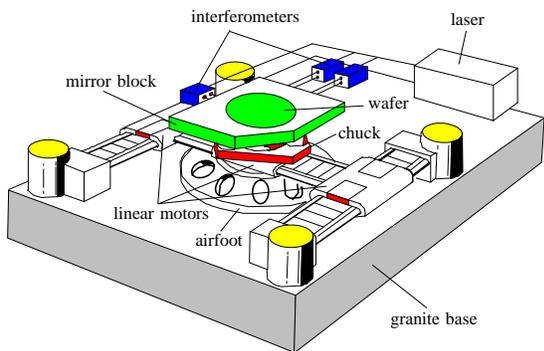
The ILC methods discussed in this paper have been applied in one direction (single input, single output) of a high precision wafer stage as shown in figure 1. This machine consists of a large granite base with a moving table on top of it (the chuck) that is actuated by linear electro-motors. The granite base is fitted with dampers to the ground to isolate it from outside disturbances. The measurement system consists of laser-interferometers which measure the position of a reflective surface on the sides of the table. This experimental setup is very similar to modern wafer stages used in IC production, which shows the relevance for industrial application.

The system under consideration in this paper is quite typical for a motion control system. The input to the system is a force and the output is the position of the system. This means that the overall system will behave like a double integrator making the system marginally stable at best. Therefore the system contains a PID feedback controller not only for disturbance suppression but also for stability.

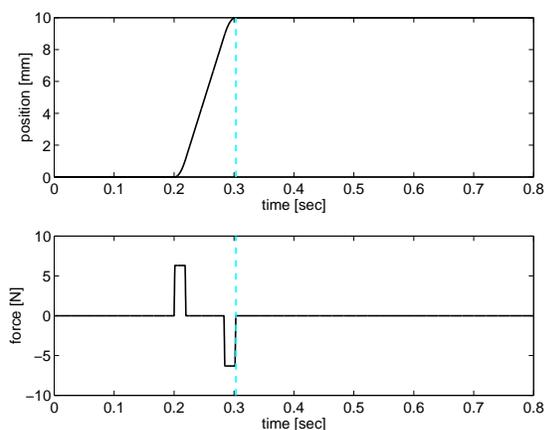
During the lithographic process of IC manufacture the chuck containing a silicon disk is repeatedly moved from one position to the next position to expose each part of the silicon disk. The trajectory considered is a

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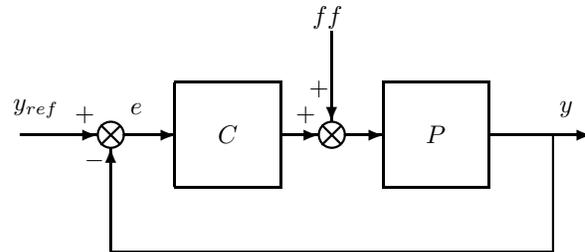
**Figure 1:** Wafer stage of a wafer stepper picture is show with courtesy of D.de Roover [1]



**Figure 2:** trajectory with force feedforward

single one of such steps of the wafer stage, a 3rd order profile based on the physical and practical limitations of the sensors and actuators. The exposure of the IC, can only occur when the errors are within certain limits after movement, so the errors *after* the step, denoted by the vertical dotted line, are of primary importance here. Figure 2 shows the reference trajectory and the force feedforward signal. The ILC will focus on adjusting the feedforward signal to obtain the smallest error possible with respect to the reference trajectory.

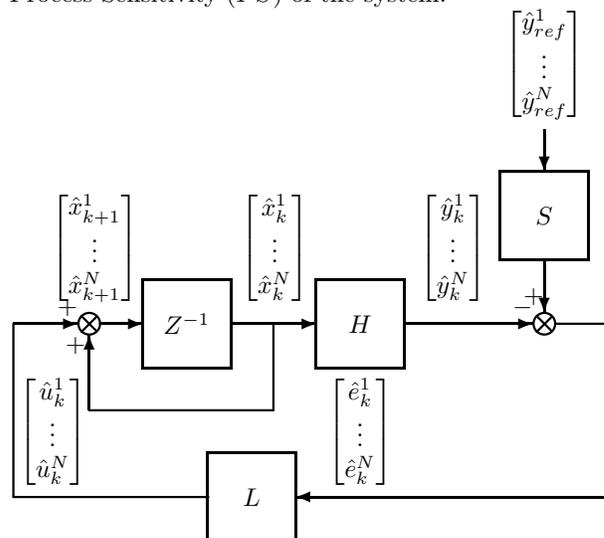
The relevant transmission path for the ILC is therefore the transfer function from the feedforward ( $ff$ ) to the output ( $y$ ), as shown in figure 3. This path will be denoted as the *process sensitivity*,  $PS = P(1 + PC)^{-1}$ . The path from the reference to the error, the *sensitivity*, will be denoted by  $S = (1 + PC)^{-1}$



**Figure 3:** Closed loop system of the wafer stage.  $P$  denotes the waferstage,  $C$  the feedback controller

### 3 General Iterative Learning Control

To clearly understand the nature of an Iterative Learning Controller, the operation of the system during a single trial is described in a lifted system setting as shown in figure 4. The input and output vectors of the system are lifted into column vectors, so that the system response can be seen as a matrix mapping,  $H$ , from input to output ([3],[4]). Note that for an LTI system, this matrix will be the impulse response matrix of the Process Sensitivity ( $PS$ ) of the system.



**Figure 4:** Lifted System ILC setting.

The principle of ILC is that the measurement of the tracking error from the current trial is used to correct the errors in the next trial ([5],[6],[7],[8]). In figure 4 this means that the error vector  $\hat{e}_k$ , based on the input  $x_k$  and reference  $y_{ref}$ , are used to obtain the input for the next trial  $x_{k+1}$ . The goal of the ILC is then to eliminate the 'disturbance' introduced by the trajectory  $y_{ref}$ . The internal model principle, [9], states that this disturbance, which is constant from trial to trial, can only be eliminated if the controller contains an integrator, shown with the feedback loop around  $Z^{-1}$ ,

which denotes 1 trial delay.  $L$  is the feedback gain matrix to stabilise the system. Any controller  $L$  that stabilises this system is a convergent ILC controller.

Note that the ideal ILC controller is  $L = H^{-1}$  which would result in a system output at trial 2 of:

$$\begin{aligned}\hat{y}_1 &= 0 \\ \hat{y}_2 &= HH^{-1}(S\hat{y}_{ref} + \hat{y}_1) \Rightarrow \\ \hat{y}_2 &= S\hat{y}_{ref}\end{aligned}\quad (1)$$

resulting in zero tracking error after 1 trial. In practice however the model of  $H$  is not entirely accurate, invertible, or both.

These ILC designs are based on the assumption that the reference,  $y_{ref}$ , and the system response,  $H$  and  $S$ , do not change from trial to trial. If there is a variation in either the reference or the system response it will be considered as a source of system noise.

#### 4 Basic ILC using memory output

The basic ILC design method discussed in this paper, is based on a combination of an inverse of the Process Sensitivity ( $PS$ ) of the plant,  $L$ , and a low-pass filter,  $Q$  as described in e.g. [1]. These filters are implemented as recursive filters (transfer functions), so they operate on a small section of the memory buffer in a recursive way.

The process sensitivity of the system needs to be inverted as accurately as possible to get good ILC performance. In this case the inverse is determined as a Zero Phase Error Tracking Controller [10]. Alternative techniques for this inverse can be found in e.g. [1], but they essentially all entail the approximation of the inverse of the process sensitivity of the plant.

Due to the nature of identification procedures, models will be inaccurate in the frequency ranges where the system gain is small. Since the system under consideration is a double integrator, the gain will drop off with a slope of -4 with increasing frequency, locating most of the model uncertainty at high frequencies. For this reason the low-pass filter,  $Q$ , is introduced to eliminate the influence of these frequencies in the ILC. This low-pass filter can be implemented in various ways, in this case however the  $Q$ -filter is placed inside the feedback loop with the trial delay  $Z^{-1}$ . This then results in the ILC recursion:

$$\begin{aligned}\hat{x}_{k+1} &= Q(\hat{x}_k + L\hat{e}_k) \\ \hat{x}_{k+1} &= Q(\hat{x}_k - LH\hat{x}_k + LS\hat{y}_{ref})\end{aligned}\quad (2)$$

which shows that the ILC will never converge to a solution with zero tracking error in the frequency range where  $Q \neq 1$ .

Using transfer functions like this on a finite time signal will introduce undesired, transient effects at the start and end of the trajectory. With this method, this effect is eliminated by extending the measurement vector on both ends and after filtering cutting off these extensions, effectively eliminating these transient effects. The method described in the next section takes this finite time behaviour into account explicitly.

#### 5 ILC using full memory states

The ILC controller  $L$  in figure 4, is the feedback interconnection between the error signal  $\hat{e}_k$  and the input of the system  $\hat{x}_k$ . The system we now consider is a multivariable system given by:

$$\begin{aligned}\hat{x}_{k+1} &= \hat{x}_k + \hat{u}_k \\ \hat{e}_k &= -H\hat{x}_k + \hat{y}_{ref} \\ \hat{u}_0 &= \hat{z}_0\end{aligned}\quad (3)$$

this clearly shows that the states of the ILC system are  $\hat{x}_k$ , and the objective of the ILC,  $L$ , will be to stabilise this system. A state feedback design method using the full ILC buffer-states is referred to as buffer-state ILC. A full buffer-state feedback controller is generally very high dimensional (square of the number of samples in the trajectory), however in [11] was shown that this high dimensional problem can be solved relatively easily. The method used in this paper uses method that is less numerically demanding, using the assumption the plant can be described as an LTI system [2], as is also assumed with basic ILC.

Any feedback design method can be used for the design of  $L$ , but in general a trade-off between the output performance and the system input (the buffer-states  $x_k$ ) is preferable. This would result in a trade-off between non systematic noise amplification and convergence (stability) of the ILC. A particularly charming method was presented in [11], where a constraint on the noise amplification and a Linear Quadratic objective are taken into consideration in a multi-objective design. For clarity, in this paper only the results for an LQR design are discussed, with a linear quadratic objective to balance the inputs and outputs of the system, resulting in the optimal controller that minimises this objective:

$$\begin{aligned}J &= \sum_{k=1}^M \hat{y}_k^T Q \hat{y}_k + \hat{u}_k^T R \hat{u}_k \\ &= \sum_{k=1}^M \hat{x}_k^T H^T Q H \hat{x}_k + \hat{u}_k^T R \hat{u}_k\end{aligned}\quad (4)$$

The solution to this problem can be found by finding the solution to the discrete time algebraic Riccati equation, but since the number of states is equal to the number of samples in the trajectory, the system tends to get

very large if long trajectories are considered which may result in a numerically intensive procedure.

Consider weighting  $Q = I$  and  $R = \beta$ :

$$J = \sum_{k=1}^M x_k^T H^T H x_k + \beta u_k^T u_k \quad (5)$$

The corresponding Riccati equation is:

$$-S(\beta I + S)^{-1}S + H^T H = 0 \quad (6)$$

with feedback interconnection:

$$L = S^{-1}H^T \quad (7)$$

The solution to the Riccati equation (6) can be approximated by:

$$S = H^T H + \beta I \quad (8)$$

Substitution of this approximate solution in the Riccati equation shows that it is a solution to an optimal control problem with slightly different weighting  $Q_{eff}$ , so it is still a solution for a stabilising (convergent) ILC:

$$Q_{eff} = \left( H^T H + \beta^2 I (H^T H + 2\beta I)^{-1} \right) \quad (9)$$

This solution can be calculated with a simulation of a Hamiltonian system, where only a state space model of the process sensitivity ( $A, B, C$ ) and the relative weighting ( $\beta$ ) is needed, which is not very numerically demanding:

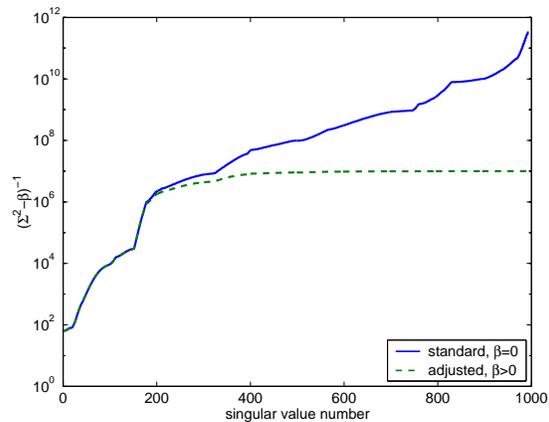
$$\begin{aligned} x(l+1) &= Ax(l) + Bu(l) \\ p(l) &= A^T p(l+1) + C^T C x(l) \\ u(l) &= \beta^{-1} y(l) - \beta^{-1} B^T p(l) \end{aligned} \quad (10)$$

For a detailed derivation and explanation of this equation please refer to [2]. This method can be used to exactly generate the solution in equation 7 without the need for solving a large dimensional Riccati equation.

To understand the influence of  $\beta$  on the ILC feedback, consider the singular value decomposition of  $H$ , where  $\mathcal{U}$  and  $\mathcal{V}$  are unitary orthogonal matrices and  $\Sigma$  contains the singular values on the main diagonal ordered from large to small:

$$H = \mathcal{U}\Sigma\mathcal{V}^T \quad (11)$$

The singular values and their corresponding singular vectors ( $\mathcal{V}$  and  $\mathcal{U}$ ) can be interpreted similarly to a Bode plot. A singular value is the gain of the system of a signal based on the corresponding singular vector. For the system under consideration, most singular vectors corresponding to small singular values will contain high frequencies, and the singular vectors corresponding to the large singular value will contain low



**Figure 5:** influence of  $\beta$  on  $L$

frequencies. This order may not hold for systems that have small gains at different frequency ranges, in which case the corresponding singular vectors will be shaped accordingly.

Substitution of equation (11) into the ILC solution (7) yields:

$$L = \mathcal{V} (\Sigma^2 + \beta I)^{-1} \Sigma \mathcal{U}^T \quad (12)$$

This illustrates that the exact inverse,  $\mathcal{V}\Sigma^{-1}\mathcal{U}^T$ , is adjusted by  $\beta$ . The singular values of  $L$  are determined by the product:  $(\Sigma^2 + \beta I)^{-1} \Sigma$ . The smallest singular values are limited in size for the inversion as shown in figure 5. Therefore the system dynamics corresponding to the small singular values will be fed back with a reduced gain, the meaning of which will become clear in later sections.

## 6 ILC experiments on the wafer stage.

This section will discuss the effects ILC has on system noise when applied to the wafer stage discussed above. The system noise under feedback without ILC is within a 30nm limit and the requirement on the system is that the error stays within a 100nm limit (shown as the grey band in the figures) after motion has occurred. First the classical ILC results are shown, followed by the state-based ILC results.

The initial trial is started with an estimate of the force profile shown in figure 2. The error for trial 1 is shown in figure 6. The error after 15 trials with basic ILC in figure 7 shows that the overall tracking error is reduced considerably at the cost of amplification of system noise to a level that is no longer acceptable. The only way to reduce this effect with basic ILC is by reducing the speed of convergence, the *learning gain* as illustrated in figure 8.

As an indication of the convergence rate, the energy

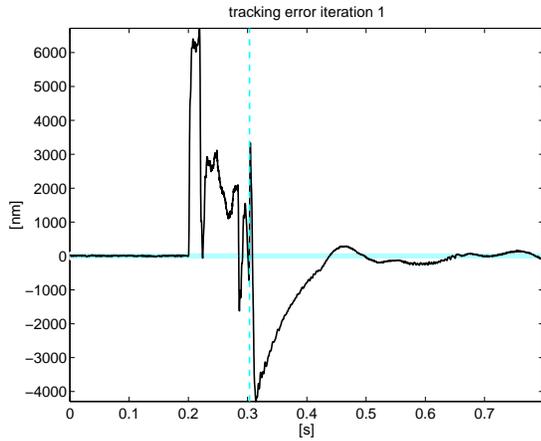


Figure 6: ILC error, trial 1

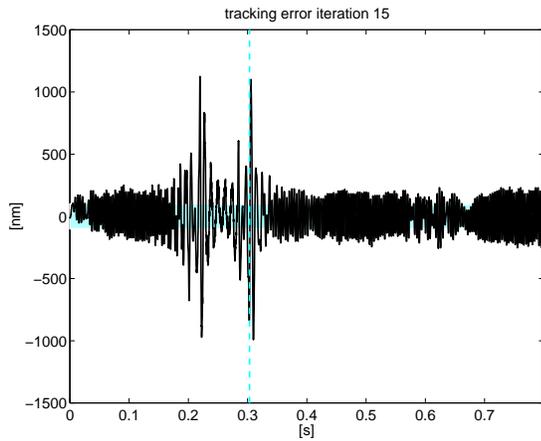


Figure 7: classic ILC error, trial 15

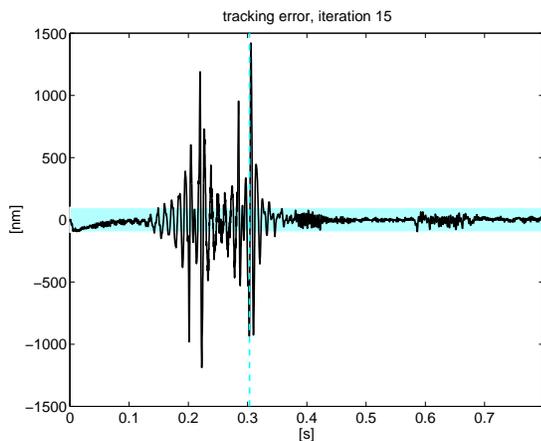


Figure 8: classic ILC error, trial 15, learning gain 0.5

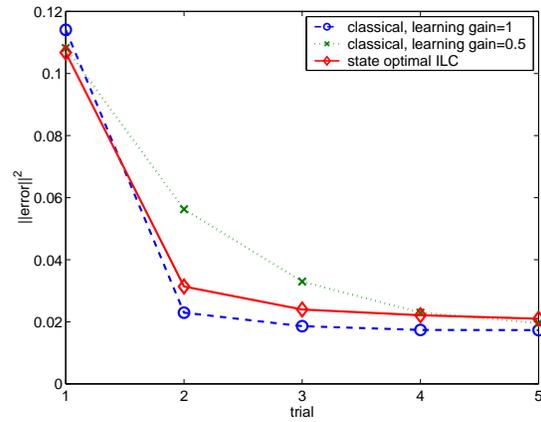


Figure 9: Error norm for first 5 trials

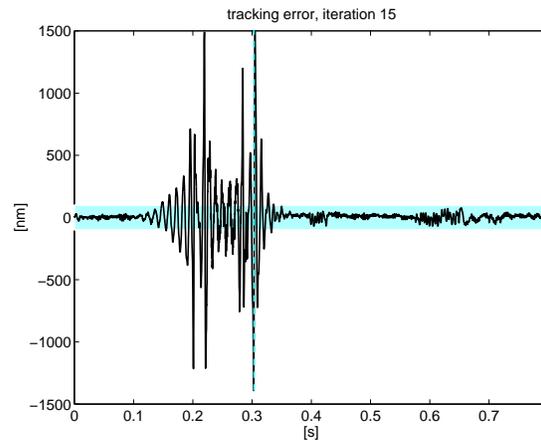
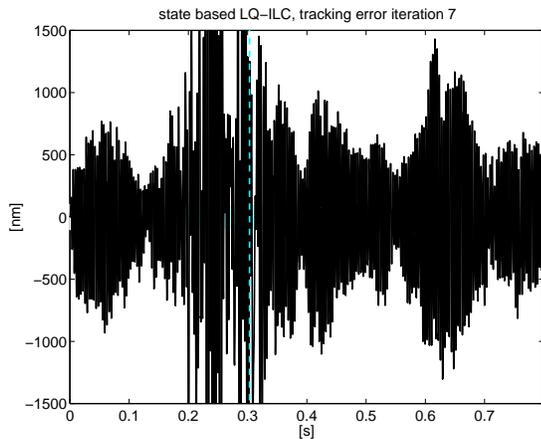


Figure 10: state based LQ-ILC error, trial 15

norm of the error signal for the first 5 trials is shown in figure 9, for basic ILC with regular and low learning gain, and also for buffer-state ILC. This illustrates the speed of convergence for the ILC designs. The state-based ILC converges about as fast as the basic ILC, but examining the results in figure 10 shows that the buffer-state ILC solution does not have the undesired amplification of system noise apparent in figure 7.

Figure 10 shows the ILC error for  $\beta = 10^{-3}$ , which gives a nice balance between tracking error performance and noise influence on the input signal. Illustrated in figure 11 is the effect of a considerably smaller value of  $\beta$ ,  $\beta = 10^{-5}$ . After 5 trials already the noise is amplified to an unacceptable level, indicating the possibilities for tuning the ILC design with parameter  $\beta$ .

As discussed above,  $\beta$  limits the ILC feedback for system dynamics corresponding to small singular values. Since the signal to noise ratio for those dynamics generally is low, this means that the system dynamics that are most influenced by noise will be handled carefully by the ILC. Effectively the buffer-state ILC design



**Figure 11:** Error norm for first 5 trials

tunes the learning gain separately for different system dynamics using the fully available buffer-state, creating an optimum balance between overall convergence and noise amplification. With basic ILC it is only possible to tune the learning gain for *all* system dynamics, or if the system dynamics can be separated into distinct frequency regions with the  $Q$  filter, which is not generally easy to do. For very small  $\beta$ , the ILC design will be more like a true inverse of the system (like basic ILC), and will thus not handle system noise as well. As such the tuning parameter  $\beta$  for state based ILC is very powerful parameter to design the behaviour of an ILC.

## 7 Conclusions

Most common ILC design methods generally only focus on convergence of the ILC, without any consideration of the effect of the ILC on system noise. As illustrated, even with very little system noise in a practical set-up, the influence of ILC on system noise can be devastating. Using a state-based ILC technique is a very easy design method that has been shown to be able to handle both ILC convergence and noise amplification very well in a practical setting.

A buffer-state ILC has been shown to converge to the final solution just as well as a basic ILC design, but the final performance with respect to system noise is considerably better. This is achieved by using a full buffer-state feedback design. The only way to achieve similar performance with respect to noise with basic ILC is by sacrificing the entire convergence rate.

The trade-off between convergence rate and noise amplification is quite common for any feedback design setting, but as yet for ILC this tradeoff has not been given much consideration. In this paper has been shown that

is practical, easy and necessary to design an ILC while considering the influence of system noise, even in system that have only very limited noise.

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